

Name: Mahyar PirayeshDate: Oct 31st 2022M10 Honours: Section 2.4 Deriving Quadratic Equations

1. Derive a quadratic equation for each parabola with the given information:

a) Vertex (0, 2) with point P (-3, 11)

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + 2$$

$$y = ax^2 + 2$$

$$11 = a(-3)^2 + 2$$

$$a = 1$$

$$y = x^2 + 2$$

b) Vertex (5, 2) with point P (3, -10)

$$y = a(x-p)^2 + q$$

$$y = a(x-5)^2 + 2$$

$$-10 = a(3-5)^2 + 2$$

$$-10 = 4a + 2$$

$$a = -3$$

$$y = -3(x-5)^2 + 2$$

c) Vertex (4, -3) with point P (6, 5)

$$y = a(x-4)^2 - 3$$

$$5 = a(6-4)^2 - 3$$

$$8 = 4a$$

$$a = 2$$

$$y = 2(x-4)^2 - 3$$

d) Vertex (0, 2) opens down, and is congruent to $y = \frac{1}{3}x^2$

$$y = ax^2 + 2$$

$$y = -\frac{1}{3}x^2 + 2$$

Since they are congruent the value of a must be the same since it dictates the "fatness" or "thinness" of the parabola.

e) Vertex (0, 2) opens up, and is congruent to $y = 0.25x^2$

$$y = ax^2 + 2$$

$$y = \frac{1}{4}x^2 + 2$$

Same reason as previous question

f) X-intercepts at 3 & -4, opens down, and is congruent to $y = \frac{1}{2}x^2$

$$y = a(x-3)(x+4)$$

$$y = a(x^2 + x - 12)$$

$$y = -\frac{1}{2}(x^2 + x - 12)$$

g) Roots at 3 & -6 with point P (2, 12) on the parabola

$$y = a(x-3)(x+6)$$

$$12 = a(2-3)(2+6)$$

$$12 = a(-1)(8)$$

$$a = -\frac{3}{2}$$

$$y = -\frac{3}{2}(x-3)(x+6)$$

$$y = -\frac{3}{2}(x^2 + 3x - 18)$$

h) Roots at $-\frac{7}{4}$ & $-\frac{3}{2}$ with point P (3, 8) on the parabola

$$y = a(x + \frac{7}{4})(x + \frac{3}{2})$$

$$8 = a(3 + \frac{7}{4})(3 + \frac{3}{2})$$

$$8 = a(\frac{19}{4})(\frac{9}{2})$$

$$8 = a \frac{171}{8}$$

$$a = \frac{8}{\frac{171}{8}} = \frac{64}{171}$$

$$y = \frac{64}{171}(x + \frac{7}{4})(x + \frac{3}{2})$$

or

$$y = a(4x+7)(2x+3)$$

$$a = \frac{8}{171} \quad y = \frac{8}{171}(4x+7)(2x+3)$$

i) Roots at 10 & $-\frac{5}{6}$ with point P (5, 10) on the parabola

$$y = a(x-10)(6x+5)$$

$$10 = a(-5)(35)$$

$$a = \frac{10^{-2}}{(-5)(35)} = -\frac{2}{35}$$

$$y = -\frac{2}{35}(x-10)(6x+5)$$

Q6

A.O.S: $x=3$

Root: 7

Root: x

$\frac{7+x}{2}=3$

$x=-1$

$y=a(x+1)(x-7)$

$y=-\frac{2}{14}(x+1)(x-7)$

j) 3 Points on the parabola: (2,6), (4,6), and (5,12)

$$y=a(x-r_1)(x-r_2)+d$$

$$y=a(x-4)(x-2)+6$$

$$12=a(5-4)(5-2)+6$$

$$12=3a+6$$

$$a=2$$

$$y=2(x-2)(x-4)+6$$

k) Maximum at 3, y-intercept at -12, and point (10,-12)

$$y=a(x-p)^2+q$$

$$y=a(x-p)^2+3$$

y-intercept = (x,y) = (0,-12)

Other point = (x,y) = (10,-12)

$$A.O.S: x=\frac{0+10}{2}=5$$

$$y=a(x-5)^2+3$$

$$-12=a(-5)^2+3$$

$$a=\frac{-15}{25}=-\frac{3}{5}$$

$$y=-\frac{3}{5}(x-5)^2+3$$

l) Axis of Symmetry $x=3$, y-intercept at 4.5, one x-intercept at 7, opens down

$$y=a(x-3)^2+q$$

$$\frac{9}{2}=a(-3)^2+q \quad \left\{ \begin{array}{l} 0=a(4)^2+q \\ 0=16a+q \end{array} \right.$$

$$\frac{9}{2}=9a+q$$

$$-\frac{9}{2}=7a$$

$$q=\frac{9}{2}+\frac{63}{14}$$

$$a=-\frac{9}{14}$$

$$q=\frac{63+81}{14}=\frac{144}{14}=\frac{72}{7}$$

$$y=-\frac{9}{14}(x-3)^2+\frac{72}{7}$$

2. Determine the value of the Discriminant and the Nature of the Roots:

a) $4x^2+10x+9=0$

$$y=ax^2+bx+c$$

$a=4$
 $b=10$
 $c=9$

$$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-10 \pm \sqrt{100-144}}{8}$$

$$\text{Discriminant} = b^2-4a = 100-144 = -44$$

No real roots

b) $-x^2+6x+7=0$

$$\text{Discriminant: } 6^2-4(-1)(7)=64$$

2 real roots

c) $-3x^2+\frac{1}{2}x+4=0$

$$\text{Discriminant} = D = \frac{1}{4}-4(-3)(4)$$

$$= \frac{1}{4}+48 = 48\frac{1}{4}$$

2 real roots

d) $5x^2-3x+\frac{1}{4}=0$

$$D=9-4(5)(\frac{1}{4})=4$$

2 real roots

e) $(x+3)^2=1$

$$x^2+6x+8=0$$

$$D=36-4(8)=4$$

2 real roots

f) $\frac{x^2}{-3}=4x$

$$x^2=-12x$$

$$x^2+12x=0$$

$$D=144-0=144$$

2 real roots

g) $200 + 33x + x^2 = 0$

$D = 33^2 - (4)(1)(200) = 289$

2 real roots

h) $0 = x^2 + 12x - 85$

$D = 12^2 - (4)(1)(-85) = 484$

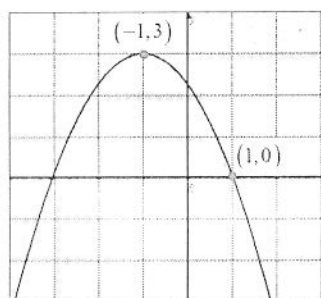
2 real roots

i) $0 = 3x^2 - 12x - 288$

$D = 144 - (4)(3)(-288) = 3600$

2 real roots

3. Derive a quadratic equation in the form of $y = a(x-p)^2 + q$ for each of the following graphs:

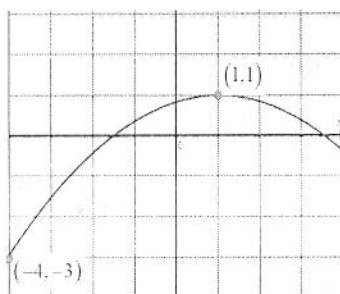


$y = a(x+1)^2 + 3$

$0 = a(2)^2 + 3$

$a = -\frac{3}{4}$

$y = -\frac{3}{4}(x+1)^2 + 3$

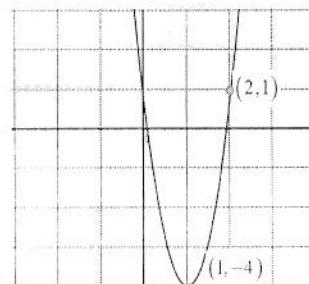


$y = a(x-1)^2 + 1$

$-3 = a(-5)^2 + 1$

$a = -\frac{4}{25}$

$y = -\frac{4}{25}(x-1)^2 + 1$



$y = a(x-1)^2 - 4$

$1 = a(1)^2 - 4$

$a = 5$

$y = 5(x-1)^2 - 4$

4. Determine the value(s) of "k" in each equation so that one root is double the other root:

a) $x^2 + kx + 50 = 0$

$y = a(x-r)(x-2r)$

$y = (x-r)(x-2r)$

$y = x^2 - 3rx + 2r^2$

$y = x^2 + kx + 50$

$2r^2 = 50$

$r^2 = \pm 5$

$-3r = k$

$k_1 = -3(-5) = 15$

$k_2 = -3(5) = -15$

b) $4x^2 + kx + 4 = 0$

$y = a(x-r)(x-2r)$

$y = 4(x^2 - 3rx + 2r^2)$

$y = 4x^2 - 12rx + 8r^2$

$y = 4x^2 + kx + 4$

$8r^2 = 4$

$r^2 = \frac{1}{2}$

$r = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$

$-12r = k$

$k_1 = -12(\frac{\sqrt{2}}{2}) = -6\sqrt{2}$ $k_2 = -12(-\frac{\sqrt{2}}{2}) = 6\sqrt{2}$

c) $\frac{2x^2}{(a=2)} - 3x + k = 0$

$y = 2(x-r)(x-2r)$

$y = 2(x^2 - 3rx + 2r^2)$

$y = 2x^2 - 6rx + 4r^2$

$y = 2x^2 - 3x + k$

$-3 = -6r$ $\begin{cases} 4r^2 = k \\ k = 4(\frac{1}{2})^2 = 1 \end{cases}$

$r = \frac{1}{2}$

5. Determine the value(s) of "k" in each equation so that one root is triple the other:

a) $3x^2 - 4x + k = 0$

b) $4x^2 + kx + 27 = 0$

c) $16x^2 + kx + 27 = 0$

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$$a) 3x^2 - 4x + k = 0$$

$$y = a(x-r)(x-3r)$$

$$a=3$$

$$y = 3(x^2 - 4x + 3r^2)$$

$$y = 3x^2 - 12rx + 9r^2$$

$$y = 3x^2 - 4x + k$$

$$-12r = -4 \Rightarrow 9r^2 = k$$

$$r = \frac{1}{3}$$

6. Determine the value(s) of "k" in each equation so that one root is equal to the other:

$$a) 4x^2 + kx + 25 = 0$$

$$y = a(x-r)^2$$

$$a=4$$

$$y = 4(x^2 - 2rx + r^2)$$

$$y = 4x^2 - 8rx + 4r^2$$

$$y = 4x^2 + kx + 25$$

$$4r^2 = 25$$

$$r^2 = \frac{25}{4}$$

$$r = \pm \frac{5}{2}$$

$$-8r = k$$

$$k = -8(\pm \frac{5}{2})$$

$$k = \pm 20$$

$$b) 4x^2 + kx + 27 = 0$$

$$y = a(x-r)(x-3r)$$

$$a=4$$

$$y = 4(x^2 - 4r + 3r^2)$$

$$y = 4x^2 - 16rx + 12r^2$$

$$y = 4x^2 + kx + 27$$

$$-16r = k$$

$$k = -16(\pm \frac{3}{2})$$

$$k = \pm 24$$

$$12r^2 = 27$$

$$r = \pm \frac{3\sqrt{3}}{2\sqrt{3}} = \pm \frac{3}{2}$$

$$c) 16x^2 + kx + 27 = 0$$

$$y = a(x-r)(x-3r)$$

$$a=16$$

$$y = 16(x^2 - 4r + 3r^2)$$

$$y = 16x^2 - 64r + 48r^2$$

$$y = 16x^2 + kx + 27$$

$$48r^2 = 27$$

$$-64r = k$$

$$r^2 = \frac{27}{48}$$

$$r = \pm \frac{3}{4}$$

$$k = -64(\pm \frac{3}{4})$$

$$k = \pm 48$$

$$c) 9x^2 - kx + 1 = 0$$

$$y = a(x-r)^2$$

$$a=9$$

$$y = 9(x^2 - 2rx + r^2)$$

$$y = 9x^2 - 18rx + 9r^2$$

$$y = 9x^2 - kx + 1$$

$$9r^2 = 1$$

$$r = \pm \sqrt{\frac{1}{9}} = \pm \frac{1}{3}$$

$$-18r = -k$$

$$k = 18(\pm \frac{1}{3}) = \pm 6$$

7. For what values of "k" will each equation have either "2 different roots", "2 equal roots", or "no roots"

$$a) 4x^2 + kx + 6 = 0 \text{ (2 Different Roots)}$$

$$\text{Roots: } ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$4x^2 + kx + 6 = 0$$

$$x = \frac{-k \pm \sqrt{k^2 - 4(4)(6)}}{8} = \frac{-k \pm \sqrt{k^2 - 96}}{8}$$

In order for the parabola to have 2 different roots, the discriminant must be positive.

$$k^2 - 96 > 0 \quad k^2 > 96$$

$$b) 6x^2 - 10x + k = 0 \text{ (2 equal roots)}$$

$$\text{Roots: } \frac{-(-10) \pm \sqrt{100 - 4(6)(k)}}{12} = \frac{10 \pm \sqrt{100 - 24k}}{12}$$

The discriminant must be equal to zero in order for the parabola to have 1 root (2 equal roots)

$$100 - 24k = 0$$

$$24k = 100$$

$$k = \frac{100}{24} = \frac{25}{6}$$

$$k = \frac{25}{6}$$

$$k > 4\sqrt{6} \text{ or } k < -4\sqrt{6}$$

$$c) (2k+1)x^2 - 3x + 5 = 0 \text{ (No Roots)}$$

$$D = \text{Discriminant: } ax^2 + bx + c = 0$$

$$\text{Discriminant} = b^2 - 4ac$$

$$D = (-3)^2 - 4(5)(2k+1) = 9 - 20(2k+1)$$

$$= 9 - 40k - 20 = -40k - 11$$

The discriminant must be negative.

$$-40k - 11 < 0$$

$$k > -\frac{11}{40}$$

$$d) (5k-1)x^2 - 4kx + 1 = 0 \text{ (2 Distinct Roots)}$$

$$D = b^2 - 4ac$$

$$D = (4k)^2 - 4(1)(5k-1) = 16k^2 - 20k + 4$$

The discriminant must be positive:

$$16k^2 - 20k + 4 > 0$$

$$16k^2 - 20k + 4 = 0$$

$$(4k-1)(4k-4) = 0$$

$$k_1 = \frac{1}{4}$$

$$k_2 = 1$$

$$k > 1 \text{ or } k < \frac{1}{4}$$

e) $kx^2 - 5x + 2k = 0$ (2 Equal Roots)

$$D = (-5)^2 - 4(2k)(k)$$

$$D = 25 - 8k^2$$

$$-8k^2 + 25 \geq 0$$

$$8k^2 - 25 \leq 0$$

$$k = \frac{-5 \pm \sqrt{25 - 4(8)(-25)}}{16} = \frac{-5 \pm \sqrt{25 + 800}}{16} = \frac{-5 \pm \sqrt{825}}{16}$$

8. $f(x)$ is a quadratic function with zeroes at $x=2$ and $x=4$. $f(0)=1$, what is $f(-1)$?

$$y = a(x-r_1)(x-r_2)$$

$$y = \frac{1}{8}(x-2)(x-4)$$

$$y = a(x-2)(x-4)$$

$$f(-1) = \frac{1}{8}(-1-2)(-1-4)$$

$$f(0)=1$$

$\Rightarrow$

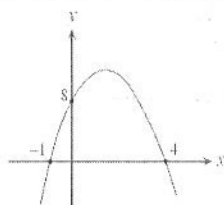
$$f(-1) = \frac{1}{8}(-3)(-5) = \frac{15}{8}$$

$$1 = a(-2)(-4)$$

$$a = \frac{1}{8}$$

$$f(-1) = \frac{15}{8}$$

9. In the diagram, the parabola has x-intercepts at -1 and 4, with the y-intercept at 8. If the parabola passes through the point (3,w), what is the value of "w"?



$$y = a(x+1)(x-4)$$

$$(x,y) = (0,8)$$

$$8 = a(1)(-4)$$

$$a = -2$$

$$y = -2(x+1)(x-4)$$

$$(x,y) = (3,w)$$

$$w = -2(3+1)(3-4) = -2(-4) = 8$$

$$w = 8$$

10. Let m and n be the roots of the equation $ax^2 + bx + c = 0$. Let $px^2 + qx + r = 0$ be a quadratic equation for which $m+2$ and $n+2$ are roots. If $p=a$, then $q+r$, expressed in terms of a , b , and c is:

$$q+r = -4a+b+c-2b+4a = c-b$$

a) $c+3b$

(b) $c-b$

c) $c+3b+8a$

d) $c-b+4a$

e) $c-b+8a$

$$0 = a(x-m)(x-n)$$

$$0 = a(x^2 - (m+n)x + mn)$$

$$0 = ax^2 - ax(m+n) + amn$$

$$b = -a(m+n) = -am - an$$

$$c = amn$$

$$0 = a(x-(m+2))(x-(n+2))$$

$$0 = a(x-m-2)(x-n-2)$$

$$0 = a(x^2 - mx - nx - 2x - mn - 2m - 2n - 4)$$

$$0 = a(x^2 - 4x - mx - nx + mn + 2m + 2n + 4)$$

$$0 = ax^2 - a(4+m+n)x + a(mn+2m+2n+4)$$

$$q = -4a - am - an \quad r = amn + 2am + 2an + 4a$$

$$q = -4a - \underbrace{am - an}_{=b}$$

$$q = -4a + b$$

$$r = amn + 2am + 2an + 4a$$

$$r = c + 2(\underbrace{am + an}_{=b}) + 4a$$

$$r = c - 2b + 4a$$

11. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. What is the number of possible values for the constant "k":

a) 0

b) 1

c) 2

d) 4

e) more than four

$$0 = x^2 - 63x + k$$

$$0 = (x-m)(x-n)$$

$$0 = x^2 - (m+n)x + mn$$

$$k = mn$$

$$m+n = 63$$

Roots: m, n

m	n
0	63
1	62
2	61
3	60
4	59

At all times, either "m" or "n" will be even, so the only set of (m,n) that works is (2,61). All other sets of (m,n) will not work b/c of the reason mentioned.

12. A grid point in the plane is a point (x, y) for which both x and y are integers. The number of grid points that lie within or on the boundary of the region bounded by the parabola $y = x^2$ and the line $y = 50$ is:

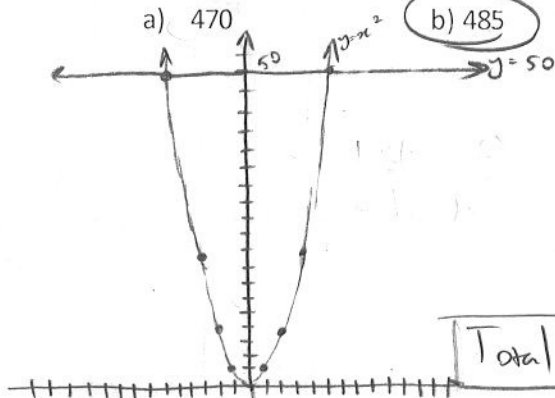
a) 470

b) 485

c) 490

d) 750

e) 765



x	y	number of points
0	0	1
1	1	3x3
2	4	(5x5)
3	9	7x7
4	16	9x9
5	25	11x11
6	36	13x13
7	49	15x15

$$\text{Total points} = 1 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2 + 13^2 + 15^2 = 485$$

13. What value(s) of k will make the following to be factored as the square of a linear polynomial?

$$4x^2 + kx + 49$$

$$y = a(x-r)^2$$

$$y = 4(x^2 - 2rx + r^2)$$

$$y = 4x^2 - 8rx + 4r^2$$

$$4r^2 = 49$$

$$-8r = k$$

$$\begin{aligned} k_1 &= -8\left(\frac{7}{2}\right) = -28 \\ k_2 &= -8\left(-\frac{7}{2}\right) = +28 \end{aligned}$$

$$r = \pm \sqrt{\frac{49}{4}} = \pm \frac{7}{2}$$

OR

Just factor

$$y = (2x-7)(2x-7)$$

$$k = -28$$

$$y = (2x+7)(2x+7)$$

$$k = 28$$

14. For what values of k does the equation $5x^2 + kx + 5 = 0$ have two different real roots?

a) $-10 < k < 10$

b) $k < -10, k > 10$

c) $k \pm 10$

d) $k > 10$

$$5x^2 + kx + 5 = 0$$

Discriminant:

$$b^2 - 4ac$$

↓

$$k^2 - 4(5)(5) = k^2 - 100$$

The discriminant has to be positive in order for there to be 2 distinct roots.

$$k^2 - 100 > 0$$

$$k^2 > 100$$

$$k > 10 \text{ or } k < -10$$

15. One root of $x^2 - kx + 18 = 0$ is twice the other. Then k = ?

$$y = a(x-r)(x-2r)$$

$$y = a(x^2 - 3rx + 2r^2)$$

$$y = x^2 - 3rx + 2r^2$$

$$y = x^2 - kx + 18$$

$$2r^2 = 18$$

$$r^2 = 9$$

$$r = \pm 3$$

$$3r = k$$

$$k = 3(\pm 3) = \pm 9$$

16. The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m , n , and p is zero. What is the value of n/p ?

a) 1

b) 2

c) 4

d) 8

e) 16

$$x^2 + px + m = 0$$

Roots: a, b

$$(x-a)(x-b) = 0$$

$$x^2 - ax - bx + ab = 0$$

$$m = ab$$

$$p = -a-b \Rightarrow p = -\frac{1}{2}m$$

$$x^2 + mx + n = 0$$

Roots: $2a, 2b$

$$(x-2a)(x-2b) = 0$$

$$x^2 - 2bx - 2ax + 4ab = 0$$

$$m = -2b - 2a$$

$$n = 4ab \Rightarrow n = 14m$$

$$\frac{n}{p} = \frac{4m}{-\frac{1}{2}m} = 8$$

17. For how many integers k do the parabolas with equations $y = -\frac{1}{8}x^2 + 4$ and $y = x^2 - k$ intersect on or above the x -axis?

a) 9

b) 32

c) 33

d) 36

e) 37

Intersection:

$$-\frac{1}{8}x^2 + 4 = x^2 - k$$

$$-\frac{9}{8}x^2 = -k - 4$$

$$\frac{9}{8}x^2 = k + 4$$

$$9x^2 = 8k + 32$$

$$x^2 = \frac{8k+32}{9}$$

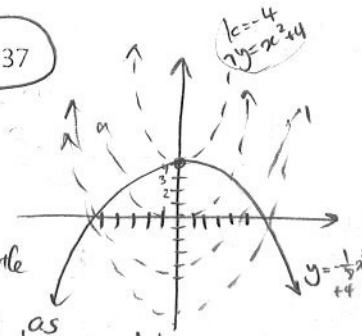
$$x = \pm \sqrt{\frac{8k+32}{9}}$$

$$y = \left(\pm \sqrt{\frac{8k+32}{9}}\right)^2 - k$$

$$y = \frac{8k+32-9k}{9} = \frac{32-k}{9}$$

If " k " exceeds 32, the " y " value of the intersection will be negative (it will be below the x -axis). " k " can also go as low as -4 since it would still intersect with the other parabola.

$$-4 \leq k \leq 32$$



18. Determine the one value of " x " such that $x - \sqrt{4x+12} = 0$. Justify your answer

$$x - \sqrt{4x+12} = 0$$

$$x = \sqrt{4x+12}$$

$$x^2 = 4x+12$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x_1 = -2$$

rej.

$$-2 \neq \sqrt{-8+12}$$

no square root can be negative.

$$x_2 = 6$$

19. Challenge: COMC 2015: A quadratic function in the form of $y = x^2 + px + q$ has one root that is double the other.

- i) If $p = -15$, then what is the value of " q "?

$$y = (x-r)(x-2r)$$

$$y = (x^2 - 3rx + 2r^2)$$

$$y = x^2 - 3rx + 2r^2$$

$$p = -3r = -15$$

$$r = 5$$

$$q = 2r^2 = 2(5)^2 = 50$$

THE QUESTION IS WRONG!!!!

$$y = x^2 + px + q$$

- ii) If one of the root is equal to 4, then what are all the possible values of $p+q$?

One root is equal to 4

$$y = x^2 + px + q$$

Roots: $(r, 0), (2r, 0)$

$$\begin{cases} r=4 \Rightarrow 2r=8 \\ 2r=4, r=2 \end{cases}$$

$y = (x-4)(x-8)$
 $y = x^2 - 12x + 32$
 $p = -12, q = 32$
 $p+q = 20$

$y = (x-2)(x-4)$
 $y = x^2 - 6x + 8$
 $p_2 = -6, q_2 = 8$
 $p_2 + q_2 = 2$

$p_1 + q_1 = 20$
 $p_2 + q_2 = 2$

- iii) If the value of $p+q=9$, then what are all the possible quadratic equations?

$$y = x^2 + px + q$$

$$y = (x-r)(x-2r)$$

$$y = x^2 - 3rx + 2r^2$$

$$p = -3r, q = 2r^2$$

$$p+q = -3r + 2r^2 = 9$$

$$2r^2 - 3r = 9$$

$$r(2r-3) = 9$$

$$r_1 = 0$$

$$r_2 = \frac{3}{2}$$

$p = -3r = -3(0) = 0, q = 2(0)^2 = 0$
 $y = x^2$

$p = -3r = -3\left(\frac{3}{2}\right) = -\frac{9}{2}, q = 2r^2 = 2\left(\frac{3}{2}\right)^2 = 2\left(\frac{9}{4}\right) = \frac{9}{2}$
 $y = x^2 - \frac{9}{2}x + \frac{9}{2}$

$y = x^2$ or $y = x^2 - \frac{9}{2}x + \frac{9}{2}$

20. Challenge: Determine all real values of "c" such that $x^2 - 4x - c - \sqrt{8x^2 - 32x - 8c} = 0$ has precisely two distinct real solutions for "x". [COMC]

$$x^2 - 4x - c = \sqrt{8x^2 - 32x - 8c}$$

$$(x^2 - 4x - c)^2 = 8(x^2 - 4x - c)$$

$$(x^2 - 4x - c)[(x^2 - 4x - c) - 8] = 0$$

Discriminant: $\downarrow$

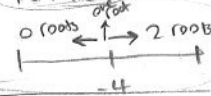
$$\sqrt{16+4c}$$

Discriminant: $\downarrow$

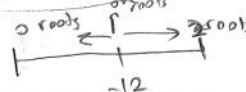
$$\sqrt{16+4c+32}$$

If we are going to have 2 roots only, one parabola needs to have 0 roots, and the other needs to have 2 roots:

$$16+4c$$



$$16+4c+32 = 4c+48$$



If $-12 < c < -4$, then the quartic function above (in the question) will have 2 roots only.